

CÁLCULO TEMA 1 - (vol III)

(M1) $\lim_{x \rightarrow 1} \frac{x^3 - 3x + 2}{x^4 - 4x + 3} = \frac{0}{0} = \lim_{x \rightarrow 1} \frac{(x-1) \cdot (x^2 + x - 2)}{(x-1)(x^3 + x^2 + x - 3)} = \frac{0}{0} = \lim_{x \rightarrow 1} \frac{(x-1)(x+2)}{(x-1)(x^2 + 2x + 3)} = \frac{3}{6} = \boxed{\frac{1}{2}}$

$$\begin{array}{r} x^3 - 3x + 2 \quad | \quad (x-1) \\ -x^3 \quad -x^2 \\ \hline x^2 - 3x + 2 \\ -x^2 \quad -x \\ \hline -2x + 2 \\ -2x + 2 \\ \hline 0 \end{array}$$

$L_4 \in \mathbb{C}$ 2' grado

Sol: $x=1$
 $x=-2$

RUFFINI: $(x^4 - 4x + 3) \rightarrow (x-1) \cdot (x^3 + x^2 + x - 3)$

$$\begin{array}{r|rrrrr} & 1 & 0 & 0 & -4 & 3 \\ 1 & & 1 & 1 & 1 & -3 \\ \hline & 1 & 1 & 1 & -3 & 0 \end{array}$$

$\hookrightarrow (x-1) \cdot (x^2 + 2x + 3)$

(p1) $\lim_{x \rightarrow 0} \frac{\arcsen(\overset{0}{x^2}) \cdot \cos(x) \cdot \operatorname{tg}(\sqrt[3]{x}) \cdot \log(1 + \sqrt[3]{x^2})}{x^3 + \operatorname{sen}(x^5)} =$

$$= \lim_{x \rightarrow 0} \frac{x^2 \cdot \cos(x) \cdot \overbrace{\sqrt[3]{x^3} = x} \cdot \sqrt[3]{x^2}}{x^3 + x^5} = \lim_{x \rightarrow 0} \frac{x^3 \cdot \cos(x)}{x^3 (1 + x^2)} = \lim_{x \rightarrow 0} \frac{\cos x}{1 + x^2} = \frac{1}{1} = \boxed{1}$$

P2

$$\lim_{x \rightarrow \infty} \left(\underbrace{\sqrt[3]{x^3 + 4x^2 - 1}}_a - \underbrace{x}_b \right) =$$

$$(a-b) \cdot (a^2 + ab + b^2) = a^3 - b^3$$

$$= \lim_{x \rightarrow \infty} \frac{\left(\sqrt[3]{x^3 + 4x^2 - 1} - x \right) \cdot \left(\sqrt[3]{(x^3 + 4x^2 - 1)^2} + x \sqrt[3]{x^3 + 4x^2 - 1} + x^2 \right)}{\left(\sqrt[3]{(x^3 + 4x^2 - 1)^2} + x \sqrt[3]{x^3 + 4x^2 - 1} + x^2 \right)} \quad (*)$$

$$= \lim_{x \rightarrow \infty} \frac{\cancel{x^3} + 4x^2 - 1 - \cancel{x^3}}{1} = \lim_{x \rightarrow \infty} \frac{4x^2 - 1}{\left(\underbrace{\sqrt[3]{(\cancel{x^3} + 4x^2 - 1)^2}}_{x^2} + \underbrace{\cancel{x} \cdot \sqrt[3]{\cancel{x^3} + 4x^2 - 1}}_{x \cdot x} + \underbrace{x^2}_{x^2} \right)} = \boxed{\frac{4}{3}}$$

Calcular el orden y parte principal:

(M1) a) $\sqrt[3]{x} \cdot \log(1+x)$ en $x_0 = 0$

$$\lim_{x \rightarrow 0} \frac{\sqrt[3]{x} \cdot \log(1+x)}{(x-0)^p} = A \Leftrightarrow \lim_{x \rightarrow 0} \frac{\sqrt[3]{x} \cdot x}{x^p} = A \Leftrightarrow \lim_{x \rightarrow 0} \frac{x^{4/3}}{x^p} = A \Leftrightarrow \boxed{p = \frac{4}{3}} \quad A = 1$$

$$PP(f) = A \cdot (x-x_0)^p = 1 \cdot (x-0)^{4/3} = \boxed{x^{4/3} = PP(f)}$$

(P1) b) $(1 - \cos(3x)) \cdot \tan(x) \cdot \log(1 + \sqrt[3]{x}) \cdot \sqrt{9+x^2}$ $x_0 = 0$

$$\lim_{x \rightarrow 0} \frac{\frac{9x^2}{2} \cdot x \cdot \sqrt[3]{x} \cdot \sqrt{9+x^2}}{x^p} = \lim_{x \rightarrow 0} \frac{9x^{10/3} \cdot \sqrt{9+x^2}}{2x^p} = A \Leftrightarrow \boxed{p = \frac{10}{3}} \quad A = \frac{9 \cdot \sqrt{9}}{2} \Rightarrow \boxed{A = \frac{27}{2}}$$

$$PP(f) = \frac{27}{2} \cdot (x-0)^{10/3} \Rightarrow \boxed{PP(f) = \frac{27}{2} x^{10/3}}$$

E2 Dominio y asíntotas:

$$f(x) = x \cdot e^{1/x} \quad \text{dom} = \mathbb{R} \setminus \{0\}$$

$$\text{A.V.} \quad \lim_{x \rightarrow 0^-} x \cdot e^{1/x} = 0 \quad ; \quad \lim_{x \rightarrow 0^+} x \cdot e^{1/x} = \lim_{x \rightarrow 0^+} \frac{e^{1/x}}{1/x} = \infty$$

$\underbrace{0 \cdot e^{-\infty}}_{\rightarrow 0} \quad ; \quad \underbrace{0 \cdot e^{+\infty}}_{\rightarrow \infty}$

A.V. en $x=0^+$

$$\text{A.H.} \quad \lim_{x \rightarrow -\infty} x e^{1/x} = -\infty$$

$-\infty \cdot e^0$

$$\lim_{x \rightarrow +\infty} x e^{1/x} = +\infty \quad \nexists \text{ A.M.}$$

$$\text{A.O.} \quad \lim_{x \rightarrow \infty} \frac{x e^{1/x}}{x} = \lim_{x \rightarrow \infty} e^{1/x} = e^0 = 1 = m //$$

A.O. $y = x+1$

$$\lim_{x \rightarrow \infty} x e^{1/x} - x = \lim_{x \rightarrow \infty} x (e^{1/x} - 1) = \lim_{x \rightarrow \infty} x \cdot \frac{1}{x} = \lim_{x \rightarrow \infty} 1 = 1 //$$

$\approx \frac{1}{x}$

E3 Orden y parte principal de $f(x) = (1 - \cos(3x)) + \log(x) + \log(1 + \sqrt[3]{x})$

$$\lim_{x \rightarrow 0} \frac{\frac{9x^2}{2} + x + \sqrt[3]{x}}{x^p} = A \Leftrightarrow \lim_{x \rightarrow 0} \left(\frac{\frac{9x^2}{2}}{x^p} + \frac{x}{x^p} + \frac{x^{1/3}}{x^p} \right) = A$$

$$\Leftrightarrow \boxed{p = \frac{1}{3} \quad A = 1}$$

$$\text{P.P.}(f) = 1 \cdot (x-0)^{1/3} = \boxed{x^{1/3} = \text{P.P.}(f)}$$

ej: $p=1$:

$$\lim_{x \rightarrow 0}$$

$$\frac{9/2 x^2}{x} + \frac{x}{x} + \frac{x^{1/3}}{x}$$

$\xrightarrow{0} \quad \xrightarrow{1} \quad \xrightarrow{\infty}$

$p = \frac{1}{3}$:

$$\frac{9/2 x^2}{x^{1/3}} + \frac{x}{x^{1/3}} + \frac{x^{1/3}}{x^{1/3}}$$

$0 + 0 + 1$