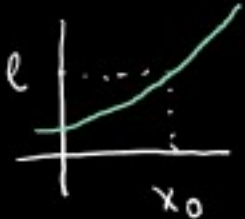


# CÁLCULO - TEMA 1.

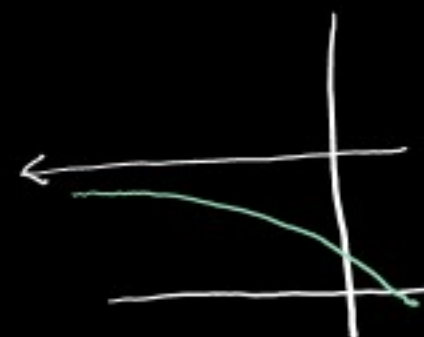
## "LÍMITES Y CONTINUIDAD DE FUNCIONES DE UNA VARIABLE REAL"

•  $\lim_{x \rightarrow x_0} f(x) = l \quad l \in \mathbb{R}$



ej:  $\lim_{x \rightarrow 0} (x^2 + 1) = 1$

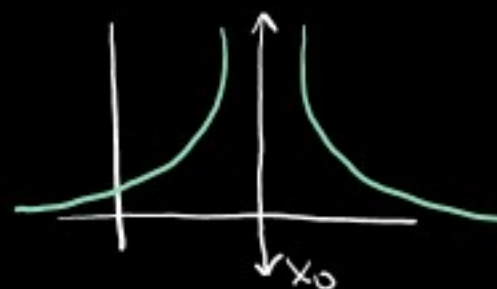
• Límite finito en el infinito:  $\lim_{x \rightarrow -\infty} f(x) = l$



$\lim_{x \rightarrow +\infty} f(x) = l$

• Límite infinito en un punto:

$\lim_{x \rightarrow x_0} f(x) = \infty$

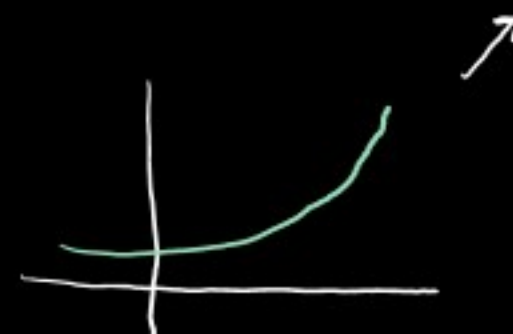


ej:  $\lim_{x \rightarrow +\infty} \frac{1}{x^2 - 3} = 0$

ej:  $\lim_{x \rightarrow 2} \frac{1}{x-2} = \infty$  (\*)

• Límite infinito en infinito:

$\lim_{x \rightarrow \infty} f(x) = \infty$



ej:  $\lim_{x \rightarrow \infty} (x^2 - 3x + 7) = +\infty$

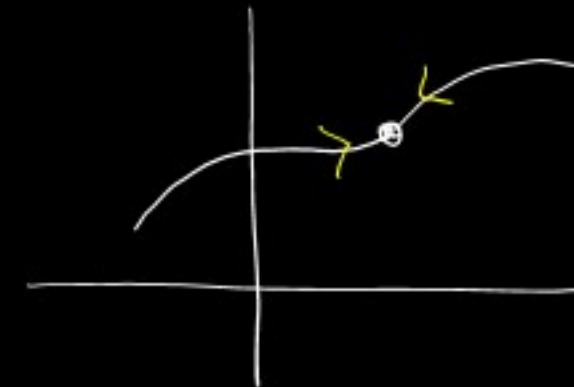
## LÍMITES LATERALES

$$\lim_{x \rightarrow x_0^+} f(x) = l^+$$

$$\lim_{x \rightarrow x_0^-} f(x) = l^-$$

$$\lim_{x \rightarrow x_0} f(x) = l \iff l^+ = l^- = l$$

$$\text{ej: } f(x) = \begin{cases} x^2 + 1 & \text{si } x < 0 \\ 1 & \text{si } x = 0 \\ 1 - \text{sen } x & \text{si } x > 0 \end{cases}$$



$$\exists \lim_{x \rightarrow 0} f(x)?$$

$$\left. \begin{aligned} \lim_{x \rightarrow 0^-} (x^2 + 1) &= 1 \\ \lim_{x \rightarrow 0^+} (1 - \text{sen } x) &= 1 - 0 = 1 \end{aligned} \right\} l^+ = l^- = l$$

$$\boxed{\lim_{x \rightarrow 0} f(x) = 1}$$



$$\begin{aligned} \cos(0) &= 1 \\ \text{sen}(0) &= 0 \end{aligned}$$

1. ASÍNTOTA VERTICAL:  $x = x_0$  es A.V.  $\Leftrightarrow \lim_{x \rightarrow x_0^-} f(x) = \pm \infty$  ó  $\lim_{x \rightarrow x_0^+} f(x) = \pm \infty$

2. ASÍNTOTA HORIZONTAL:  $y = y_0$  es A.H.  $\Leftrightarrow \lim_{x \rightarrow \pm \infty} f(x) = y_0$  ó  $\lim_{x \rightarrow \pm \infty} f(x) = y_0$

3. ASÍNTOTA OBLICUA:  $y = mx + n$  ( $m \neq 0$ ) es A.O.  $\Leftrightarrow$

1.  $\lim_{x \rightarrow \pm \infty} f(x) = \infty$

(No compatible con A.H. !!)

2.  $\lim_{x \rightarrow \infty} \frac{f(x)}{x} = m$

(Arriba un grado más que abajo)

3.  $\lim_{x \rightarrow \infty} (f(x) - mx) = n$

## Ejemplos Asintotas:

EJ1:  $f(x) = 1 - e^{-x^2}$

¿A.V.

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$$1 - \frac{1}{\infty} = 1 - 0$$

A.H.?

$$\lim_{x \rightarrow \pm \infty} 1 - e^{-x^2} \rightarrow \pm \infty$$

$$= \lim_{x \rightarrow \pm \infty} 1 - \frac{1}{e^{x^2}}$$

↙

$$= 1$$

$$= 1$$

$$\Rightarrow \boxed{\text{A.H. } y=1}$$

$$(\Rightarrow \exists \text{ A.O.})$$

EJ2:  $g(x) = \frac{x^2+1}{x+2}$

(ambos grado 2, abajo grado 1)

A.V.

⚠ comprobar cuando se anula

ej:

$$\frac{x^2-4}{x-2}$$

1. PIENSO A.V.  $x=2$  ⚠

$$\text{si } x=2 \quad \frac{0}{0}$$

$$\left( \text{si } x=-2: \frac{5}{0} \checkmark \right)$$

$$\boxed{\text{A.V. en } x=-2}$$

$$\frac{(x+2)(x-2)}{x-2} = x+2 \quad \text{¿AV!!!}$$

A.O.

$$\lim_{x \rightarrow -\infty} f(x) = -\infty$$

$$\lim_{x \rightarrow +\infty} f(x) = +\infty$$

$$\lim_{x \rightarrow \infty} \left( \frac{\frac{x^2+1}{x+2}}{x} \right) =$$

$$\lim_{x \rightarrow \infty} \frac{x^2+1}{x(x+2)} =$$

$$\lim_{x \rightarrow \infty} \frac{x^2+1}{x^2+2x} = 1 = m //$$

$$\boxed{\text{A.O. } y = x-2}$$

$$\lim_{x \rightarrow \infty} \frac{x^2+1}{x+2} - x =$$

$$\lim_{x \rightarrow \infty} \frac{x^2+1-x^2-2x}{x+2} = -2 = n //$$

# ÁLGEBRA DE LÍMITES

sean  $f(x)$  y  $g(x)$  funciones tal que  $\lim_{x \rightarrow x_0} f(x) = l_1$   $\lim_{x \rightarrow x_0} g(x) = l_2$   $l_1, l_2 \in \mathbb{R}$  :

$$1. \lim_{x \rightarrow x_0} (f(x) \pm g(x)) = l_1 \pm l_2$$

$$3. \lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = \frac{l_1}{l_2} \quad \text{si } l_2 \neq 0$$

$$2. \lim_{x \rightarrow x_0} (f(x) \cdot g(x)) = l_1 \cdot l_2$$

$$4. \lim_{x \rightarrow x_0} f(x)^{g(x)} = l_1^{l_2} \quad \text{si } l_1 > 0$$

Indeterminaciones:

$\infty - \infty$

$0 \cdot \infty$

$\frac{0}{0}$

$\frac{\infty}{\infty}$

$1^\infty$

$\infty^0$

$0^0$

$(a+b)(a-b) = a^2 - b^2$

$$ej: \lim_{x \rightarrow \infty} (\sqrt{x^2 + 8x - 3} - x) = \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2 + 8x - 3} - x)(\sqrt{x^2 + 8x - 3} + x)}{(\sqrt{x^2 + 8x - 3} + x)} \quad \checkmark$$

$$= \lim_{x \rightarrow \infty} \frac{x^2 + 8x - 3 - x^2}{\sqrt{x^2 + 8x - 3} + x} \quad \downarrow \quad 4 //$$

$$\frac{8x}{2x}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{8x}{x} - \frac{3}{x}}{\sqrt{\frac{x^2}{x^2} + \frac{8x}{x} - \frac{3}{x}} + \frac{x}{x}} = \frac{8}{1+1} = 4 //$$